Factoring

Sum or Difference of Two Cubes

Let’s Recall

The cube of an integer or an expression is a perfect cube.

For example, 8 is a perfect cube because $(2)^3$ or (2)(2)(2) = 8

x^3 is a perfect cube because x • x • x = x^3

Complete the following cubes

- $1^3 = 1 \cdot 1 \cdot 1 = 1$
- $3^3 = 3 \cdot 3 \cdot 3 = 27$
- $4^3 = 4 \cdot 4 \cdot 4 = 64$
- $6^3 = 6 \cdot 6 \cdot 6 = 216$
- $10^3 = 10 \cdot 10 \cdot 10 = 1000$

Complete the following cubes

- $(x^2)^3 = x^2 \cdot x^2 \cdot x^2 = x^{2+2+2} = x^6$
- $(y^3)^3 = y^3 \cdot y^3 \cdot y^3 = y^{3+3+3} = y^9$
- $(2xy)^3 = 2xy \cdot 2xy \cdot 2xy = 2^3x^3y^3 = 8x^3y^3$
- $(0.2x)^3 = 0.2x \cdot 0.2x \cdot 0.2x = (0.2)^2x^3 = 0.008x^3$
- $(\frac{1}{3}x)^3 = \frac{1}{3}x \cdot \frac{1}{3}x \cdot \frac{1}{3}x = (\frac{1}{3})^3x^3 = (\frac{1^3}{3^3})x^3 = \frac{1}{27}x^3$

Factoring Sum & Difference of Two Cubes

The sum and difference of two cubes can be factored using this pattern:

For any real numbers a and b

$$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$$

Or

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Let's Try

Remember

$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example 1

Factor $x^3 + 8$

a = x

b = 2

$x^3 + 2^3 = (a + b)(a^2 – ab + b^2)$

$x^3 + 2^3 = (x + 2)[(x)^2 + (x)(2) + (2)^2]$

$x^3 + 2^3 = (x + 2)(x^2 – 2x + 4)$

$= (x + 2)(x^2 – 2x + 4)$

Set Up

Substitute

Solve

Let's Try

Remember

$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example 2

Factor $27x^3 - y^6$

$a = 3x$

$b = y^2$

$(3x)^3 - (y^2)^3 = (a - b)(a^2 + ab + b^2)$

$(3x)^3 - (y^2)^3 = (3x - y^2)[(3x)^2 + (3x)(y^2) + (y^2)^2]$

$(3x)^3 - (y^2)^3 = (3x - y^2)(9x^2 + 3xy^2 + y^4)$

$= (3x - y^2)(9x^2 + 3xy^2 + y^4)$

Let's Try

Remember

$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Example 3

Factor $(xz)^3 + y^9$

$a = xz$

$b = y^3$

$(xz)^3 + (y^3)^3 = (a + b)(a^2 - ab + b^2)$

$(xz)^3 + (y^3)^3 = (xz + y^3)[(xz)^2 - (xz)(y^3) + (y^3)^2]$

$(xz)^3 + (y^3)^3 = (xz + y^3)(x^2z^2) + xzy^3 + y^6)$

$= (xz - y^3)(x^2z^2 + xzy^3 + y^6)$

Let's Apply

Remember

$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Complete the factored form for each product.

$x^3 + 64 = (x + 4)(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)$

a = x

b = 4

$x^3 + 64 = (a + b)(a^2 – ab + b^2)$

$= (x + 4)[(x)^2 - (x)(4) + (4)^2]$

$= (x + 4)(x^2 – 4x + 16)$

$(x^2 – 4x + 16)$

Let's Apply

Remember

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Complete the factored form for each product.

$8a^3 + b^3 = (2a + b)(\_\_\_\_\_\_\_\_)$

a = 2a

b = b

$(2a)^3 + (b)^3 = (a + b)(a^2 - ab + b^2)$

$= (2a + b)[(2a)^2 - (2a)(b) + (b)^2]$

$= (2a + b)(4a^2 - 2ab + b^2)$

$(4a^2 - 2ab + b^2)$

Let's Apply

Remember

$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Complete the factored form for each product.

$27x^3y^3 – 343 = (3xy - 7)(\_\_\_\_\_\_\_\_\_\_)$

a = 3xy

b = 7

$(3xy)^3 - (7)^3 = (a - b)(a^2 + ab + b^2)$

$= (3xy - 7)[(3xy)^2 + (3xy)(7) + (7)^2]$

$= (3xy - 7)(9x^2y^2 + 21xy + 49)$

$(9x^2y^2 + 21xy + 49)$

Let's Apply

Remember

$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Complete the factored form for each product.

$0.027r^3 - 0.064s^3 = (0.3r - 0.4s)(\_\_\_\_\_\_\_\_\_\_\_\_)$

$(0.3r)^3 - (0.4s)^3 = (a - b)(a^2 + ab + b^2)$

a = 0.3r

b = 0.4s

$(0.3r)^3 - (0.4s)^3 = (0.3r - 0.4s)[(0.3r)^2 + (0.3r)(0.4s) + (0.4s)^2]$

$= (0.3r - 0.4s)(0.09r^2 + 0.12rs + 0.16s^2)$

$(0.09r^2 + 0.12rs + 0.16s^2)$

Let's Apply

Remember

$a^3 + b^3 = (a + b)(a^2 – ab + b^2)$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Complete the factored form for each product.

$$\frac{1}{27}w^3 - \frac{1}{8}s^3 = (\frac{1}{3}w - \frac{1}{2}s)(\_\_\_\_\_\_\_\_\_\_\_\_)$$

$a = \frac{1}{3}w$

$b = \frac{1}{2}s$

$$(\frac{1}{3}w)^3 - (\frac{1}{2}s)^3 = (a - b)(a^2 + ab + b^2)$$

$$(\frac{1}{3}w)^3 - (\frac{1}{2}s)^3 = (\frac{1}{3}w - \frac{1}{2}s)\left[(\frac{1}{3}w)^2 + (\frac{1}{3}w)(\frac{1}{2}s) + (\frac{1}{2}s)^2\right]$$

$$= (\frac{1}{3}w - \frac{1}{2}s)(\frac{1}{9}w^2 + \frac{1}{6}ws + \frac{1}{4}s^2)$$

$$(\frac{1}{9}w^2 + \frac{1}{6}ws + \frac{1}{4}s^2)$$

Let's Analyze

Factor each polynomial completely.

$8x^3 - 125$

$(2x)^3 - (5)^3$

$(2x)^3 - (5)^3 = (a - b)(a^2 + ab + b^2)$

$(2x)^3 - (5)^3 = (2x - 5)((2x)^2 + (2x)(5) + (5)^2)$

$= (2x - 5)(4x^2 + 10x + 25)$

$x^3y^6 + 512$

$(xy^2)^3 + (8)^3$

$(xy^2)^3 + (8)^3 = (a + b)(a^2 - ab + b^2)$

$(xy^2)^3 + (8)^3 = (xy^2 + 8)((xy^2)^2 - (xy^2)(8) + (8)^2)$

$= (xy^2 + 8)(x^2y^4 - 8xy^2 + 64)$

$343m^3 - 1000n^3$

$(7m)^3 - (10n)^3$

$(7m)^3 - (10n)^3 = (a - b)(a^2 + ab + b^2)$

$(7m)^3 - (10n)^3 = (7m - 10n)((7m)^2 + (7m)(10n) + (10n)^2)$

$= (7m - 10n)(49m^2 + 70mn + 100n^2)$